Where the heck were we?

Hypothesis Testing I:

The One-Sample Case

This week Hypothesis Testing II:

Chapter 8: The Two-Sample Case

End of today: a brief review for "mid term"

Friday (review for mid term test: normal tutorial hours & locations)

9-1

Chapter 7: Last week

Hypothesis Testing:

The One-Sample Case

Compare a sample statistic with a population parameter

We take a sample of Brock students; calculate a statistic (mean GPS),

& then ask: do they differ significantly from all students in Ontario (the population parameter)?

TODAY: Chapter 8:

Hypothesis Testing II:

The Two-Sample Case

Compare a sample statistic with another sample statistic

Eg. We take a "sample of Brock students"... calculate their "mean GPA"

We take a "sample of Kings students"... calculate their "mean GPA"

Do the two sample differ significantly?

9-2

In this presentation you will learn about:

- The basic logic of the two sample case.
- Hypothesis Testing with
  Sample Means (Large Samples),
  Sample Means (Small Samples)
  Sample Proportions (Large Samples)

9-3

Example:

- Do middle- and working-class persons differ in their use of email?
- The data below report the average number of times per day that people check their email in two random samples (one of middle class individuals and the other working class individuals):

E-mail Messages

Sample 1 (Middle Class)	Sample 2 (Working Class)
$\bar{X}_1 = 8.7$	$\bar{X}_2 = 5.7$
$s_1 = 0.3$	$s_2 = 1.1$
$N_1 = 89$	$N_2 = 55$

- The middle class seem to check their email more than the working class, but is the difference *significant*?

9-4

Hypothesis Test for Two Samples: Basic Logic

We begin with a difference between sample statistics (means).

The question we test:
"Is the difference between the samples large enough to allow us to conclude (with a known probability of error) that the populations represented by the samples are different?"

The Null hypothesis H_0
There is no difference between the parameters of the two populations. $H_0: \mu_1 = \mu_2$

The Alternate hypothesis H_1:
There is a significant difference between the populations.

$H_1: \mu_1 \neq \mu_2$ *or* $H_1: \mu_1 > \mu_2$ *or* $H_1: \mu_1 < \mu_2$

Changes from One- to Two-Sample Case

- **Step 1:** in addition to samples selected according to EPSEM principles, samples must be selected independently: **Independent random sampling**.
- **Step 2:** null hypothesis statement will say the two populations are not different.
- **Step 3:** sampling distribution refers to ***difference between*** **the sample statistics**.
- **Step 4:** In computing the test statistic, we use *Z*(obtained) or *t*(obtained) with slightly revised formula, depending on the size of our sample (forthcoming)
- **Step 5:** same as before: If the test statistic, *Z*(obtained) or *t*(obtained), falls into the critical region, as marked by *Z*(critical) or *t*(critical), reject the H_0.

9-6

NOTE: STEP 4 USES DIFFERENT FORMULA!!!

- **Step 4:** In computing the test statistic, we use *Z*(obtained) or *t*(obtained) with slightly revised formula, depending on the size of our sample (forthcoming)

We will work with 3 options & 3 sets of formulae

1. If comparing sample means (2 large samples)
   1a. With population standard deviations
   1b. With only sample standard deviations

2. If comparing sample means (small samples: n_1 and $n_2 < 100$)

3. If comparing sample proportions (large samples)

9-7

1a. If comparing sample means (2 large samples) with σ

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{x}-\bar{x}}} \quad \text{with} \quad \sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

1b. If comparing sample means (2 large samples) with s

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{x}-\bar{x}}} \quad \text{with} \quad \sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}$$

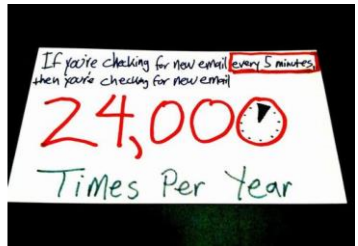
2. If sample means (small samples)

$$t(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{x}-\bar{x}}} \quad \text{with} \quad \sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

3. If sample proportions (large samples)

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}} \qquad \sigma_{p-p} = \sqrt{P_u(1-P_u)}\sqrt{\frac{n_1 + n_2}{n_1 n_2}} \qquad P_u = \frac{n_1 P_{s1} + n_2 P_{s2}}{n_1 + n_2}$$

9-8



"Psst! Buddy – you wanna check your email?"

9-9

Example:

- Do middle- and working-class persons differ in their use of email?
- The data below report the average number of times per day that people check their email in two random samples (one of middle class individuals and the other working class individuals):

E-mail Messages	
Sample 1 (Middle Class)	Sample 2 (Working Class)
$\bar{X}_1 = 8.7$	$\bar{X}_2 = 5.7$
$s_1 = 0.3$	$s_2 = 1.1$
$N_1 = 89$	$N_2 = 55$

- The middle class seem to check their email more than the working class, but is the difference *significant*?

9-10

Testing Hypotheses: The Five Step Model

1. Make assumptions and meet test requirements.
2. State the H_0.
3. Select the Sampling Distribution and Determine the Critical Region.
4. Calculate the test statistic.
5. Make a Decision and Interpret Results.

9-11

Return to our example:

E-mail Messages

Sample 1 (Middle Class)	Sample 2 (Working Class)
$\bar{X}_1 = 8.7$	$\bar{X}_2 = 5.7$
$s_1 = 0.3$	$s_2 = 1.1$
$N_1 = 89$	$N_2 = 55$

9-12

Step 1: Make Assumptions and Meet Test Requirements

- Model:
  - Independent Random Samples
    - The samples must be independent of each other (i.e. the selection of cases in the first sample has no bearing on the selection of cases in the second)
  - Level of Measurement is Interval-Ratio
    - Number of email messages -> can work with our means
  - Sampling Distribution's shape
    - N = (85+55 =144) cases which is > 100 so we can assume a normal shape.

9-13

Step 2: State the Null Hypothesis

- No direction for the difference has been predicted, so a two-tailed test is called for, as reflected in the research hypothesis:
  - H_0: $\mu_1 = \mu_2$
    - The Null asserts there is no significant difference between the populations (the two populations represented by our samples are equally likely to be using email)
  - H_1: $\mu_1 \neq \mu_2$
    - The research hypothesis contradicts the H_0 and asserts there is a significant difference between the populations.

9-14

Step 3: Select Sampling Distribution and Establish the Critical Region

- Sampling Distribution = *Z* distribution
- Alpha (α) = 0.05
- note: unless otherwise stated, use 0.05 in all significance tests (i.e. the default in most tests)
- With two tailed test: *Z* (critical) = ± 1.96

Step 4: Compute the Test Statistic

With two sample tests, use the appropriate formula (below) to compute the obtained *Z* score:

$$Z(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}}$$

The denominator in this formula is the standard deviation of the sampling distribution (i.e. the standard error)

Step 4 *(continued)*

NOTE: How do we calculate this standard error that enters into the denominator of Z(obtained)? When the population standard deviations are <u>known</u>, we use the following formula:

$$\sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

but when we only have the sample standard deviations, we use the following:

$$\sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}$$

i.e. we substitute *s* as an estimator of σ, suitably corrected for the bias (*n* is replaced by *n-1* to correct for the fact that *s* is a biased estimator of σ).

Again, the above formula only apply if the combined size of the two samples is at least N> 100

In this example: compute the Test Statistic

E-mail Messages

Sample 1 (Middle Class)	Sample 2 (Working Class)
$\bar{X}_1 = 8.7$	$\bar{X}_2 = 5.7$
$s_1 = 0.3$	$s_2 = 1.1$
$N_1 = 89$	$N_2 = 55$

We have the "sample standard deviations",..

So: calculate standard error (population standard deviations unknown):

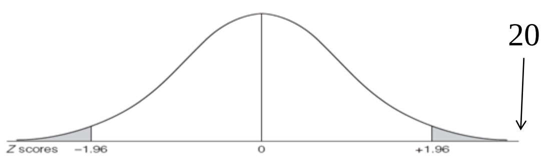
$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}} = \sqrt{\frac{.3^2}{89-1} + \frac{1.1^2}{55-1}} = \sqrt{.001 + .022} = .15$$

On this basis, you can calculate Z (obtained) with the standard error in the denominator

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}} = \frac{8.7 - 5.7}{.15} = 20$$

Step 5: Make Decision and Interpret Results

The obtained test statistic (Z = 20) falls in the Critical Region so *reject* the null hypothesis.



- The difference between the sample means is so large that we can conclude, at $\alpha = 0.05$, that a difference exists between the populations represented by the samples.
- The difference between email usage of middle- and working-class individuals *is significant.*

Hypothesis Test for Two-Sample Means: Student's *t* distribution

(Small Samples)

Hypothesis Test for Two-Sample Means: Student's *t* distribution

(Small Samples)

- For small samples (combined *N's<100), s* is too unreliable an estimator of *σ* so do not use standard normal distribution. Instead we use Student's *t* distribution.
- The formula for computing the test statistic, *t*(obtained), is:

FORMULA 8.6

$$t(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{x}-\bar{x}}}$$

where $\sigma_{\bar{x}-\bar{x}}$ is defined as:

FORMULA 8.5

$$\sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

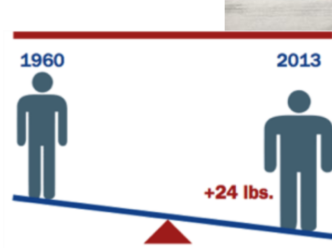
Hypothesis Test for Two-Sample Means: Student's *t* distribution

(continued)

- The logic of the five-step model for hypothesis testing is followed, using the *t* table, Appendix B, where the degrees of freedom (df) = $N_1 + N_2 - 2$.

9-21

Example: Research on Obesity,.. How to deal with the problem?



9-22

Example:

Studying "weight loss" strategies:

- 1st sample – combined cardio (30 minutes a day & weight training 30 minutes a day)

  Mean weight loss: 20 pounds
  s =5
  Sample size (n_1 = 29)

2nd sample – Solely cardio (45 minutes a day)

Mean weight loss: 18 pounds
s = 4
Sample size (n_2 = 33)

Is there a significant difference between the two??

9-23

Step 1: Make Assumptions and Meet Test Requirements

- Model:
  - Independent Random Samples
  - Level of Measurement is Interval-Ratio
    - Weight loss-> can work with our means
  - Sampling Distribution's shape
    - *N* = (29+33=62) cases which is less than 100 so we must work with t distribution

9-24

Step 2: State the Null Hypothesis

- No direction for the difference has been predicted, so a two-tailed test is called for, as reflected in the research hypothesis:
  - H_0: $\mu_1 = \mu_2$
    - The Null asserts there is no significant difference in the weight loss for the two populations
  - H_1: $\mu_1 \neq \mu_2$
    - The research hypothesis contradicts the H_0 and asserts there is a significant difference in weight loss

9-25

Step 3: Select Sampling Distribution and Establish the Critical Region

- Sampling Distribution = *t* distribution
- Alpha (α) = 0.05
- note: unless otherwise stated, use 0.05 in all significance tests (i.e. the default in most tests) df = $n_1 + n_2 - 2 = 60$
- With two tailed test: *t* (critical) = ? (from Appendix B)

Appendix B Distribution of *t*

Degrees of Freedom (df)	Level o[illegible]ificance for One-tailed Test					
	.10	.05	[illegible]	.01	.005	.0005
	Level of Significance for Two-tailed Test					
	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	[illegible]	2.771	3.690
28	1.313	1.701	2.048	[illegible]	2.763	3.674
29	1.311	1.699	2.045	[illegible]	2.756	3.659
30	1.310	1.697	2.042	[illegible]	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Source: Table III of Fisher & Yates: *Statistical Tables for Biological, Agricultural and Medical Research*, published by Longman Group Ltd., London (1974), 6th edition (previously published by Oliver & Boyd Ltd.,

9-27

Step 3: Select Sampling Distribution and Establish the Critical Region

- Sampling Distribution = *t* distribution
- Alpha (α) = 0.05
- note: unless otherwise stated, use 0.05 in all significance tests (i.e. the default in most tests) df = $N_1 + N_2 - 2 = 60$
- With two tailed test: *t* (critical) = ± 2.00 (from Appendix B)

Step 4: Compute the Test Statistic

With two sample tests, use the appropriate formula (below) to compute the obtained *t* score:

$$t(\text{obtained}) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{x}-\bar{x}}}$$

↑

BUT: must first calculate the denominator (SE)

Step 4 *(continued)*

NOTE: How do we calculate this standard error ?

When the population standard deviations are <u>unknown</u>, we use Formula 8.5 to calculate $\sigma_{\bar{x}-\bar{x}}$:

Again, the above formula only apply if the combined size of the two samples is less than 100

FORMULA 8.5

$$\sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

$$\sigma_{\bar{X}-\bar{X}} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{n_1 + n_2}{n_1 n_2}} = \sqrt{\frac{(29)(5)^2 + (33)(4)^2}{29 + 33 - 2}}\sqrt{\frac{29 + 33}{(29)(33)}} =$$

$$= 1.16$$

In this example: compute the Test Statistic

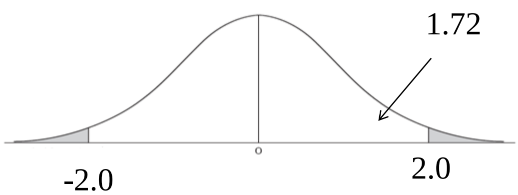
On this basis, you can calculate t (obtained) with the standard error in the denominator

$$t(obtained) = \frac{(\bar{X}_1 - \bar{X}_2)}{\sigma_{\bar{X}-\bar{X}}} = \frac{20 - 18}{1.16} = 1.72$$

9-30

Step 5: Make Decision and Interpret Results

The obtained test statistic (t = 1.72) does not fall in the Critical Region so we can not *reject* the null hypothesis.
Recall: t(critical) +/- 2.0



- The difference between the sample means is not large enough that we can
- Fail to reject the null hypothesis
- conclude, at α = 0.05, that a difference DOES NOT exist between the populations represented by the samples.
-
- The difference between the two populations using the different exercise regimes *is NOT significant.*

9-31

TWO sample test with Proportions (or percentages)….

We conduct research on educational outcomes

AFN's National Chief, Perry Bellegarde has urged the Trudeau Government to act on "education"!!

9-32

Example:

Sample from Non-Indigenous Population (N=60)
Ps1 = .23 (23 % are university educated)

Sample from Indigenous Population (N=72)
Ps2 = .10 (10% are university educated)

Are Non-Indigenous Canadians significantly more likely than Indigenous Canadians to have a university degree?

Problem here: can we infer from our samples, that are not that large?

Formula for Hypothesis Testing with Sample Proportions (Large Samples)

- Formula for proportions:

$$Z(obtained) = \frac{P_{s1} - P_{s2}}{\sigma_{p-p}}$$

Where P_{s1} is the proportion associated with the first sample, and P_{s2} is the proportion associated with the second.

- See next slide for how to calculate the denominator in this equation (*standard error*)* and the "pooled estimate of the population proportion"*….
- *Note that you need to calculate both these values in order to solve the denominator of the above equation!

To obtain standard error, most first calculate something called: P_u (the Pooled Estimate of the Population Proportion)

- To calculate P_u (the pooled estimate):

$$P_u = \frac{n_1 P_{s1} + n_2 P_{s2}}{n_1 + n_2}$$

- Which is then inserted into the following equation for the standard deviation of the sampling distribution (standard error):

$$\sigma_{p-p} = \sqrt{P_u(1 - P_u)} \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

Which then enters into the aforementioned formula for our test statistic Z(obtained)

Again, use the basic 5 step model in testing for significance…

Step 1.
Model has independent random samples,
Level of measurement is “nominal” -> work with proportions
Sampling distribution can be considered normal since N> 100

Step 2. State null hypothesis: direction? Yes, one tailed test

H_0: $P_{\mu 1} = P_{\mu 2}$
The Null asserts there is no significant difference in the proportion with a university degree for the two populations

H_1: $P_{\mu 1} > P_{\mu 2}$
The research hypothesis contradicts the H_0 and asserts there is a significant difference: Non-Aboriginal people have a higher education.. Than Aboriginal Canadians..

Step 3.

Select the sampling distribution and establish critical region

Sampling distribution is the Z distribution

Alpha is .05 one tailed

Appendix A table indicates Z(critical) = 1.65

Step 4. Calculate the test statistic

Start with "pooled estimate on the proportion"

$$P_u = \frac{n_1 P_{s1} + n_2 P_{s2}}{n_1 + n_2}$$

$$P_u = \frac{(60)(.23) + (72)(.10)}{60 + 72} = .159$$

Next: get our standard error

$$\sigma_{p-p} = \sqrt{\mathrm{P}_u(1 - \mathrm{P}_u)}\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

$$\sigma_{p-p} = \sqrt{.159(1 - .159)}\sqrt{\frac{60 + 72}{(60)(72)}} = 0.064$$

Step 4 (continued)

Then obtain your test statistic:

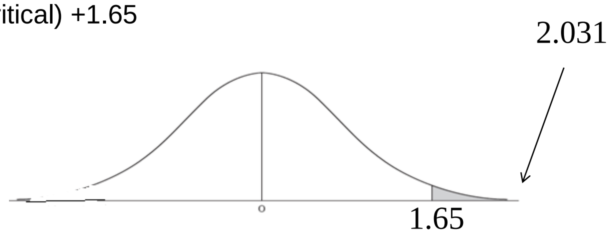
$$Z(obtained) = \frac{\mathrm{P}_{s1} - \mathrm{P}_{s2}}{\sigma_{p-p}}$$

$$Z(obtained) = \frac{.23 - .10}{.064} = 2.031$$

Step 5: Make Decision and Interpret Results

The obtained test statistic Z = 2.031) falls in the Critical Region so we can *reject* the null hypothesis.

Recall: Z(critical) +1.65

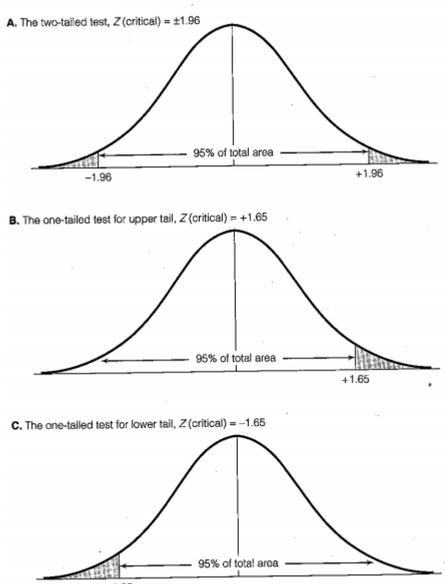


- The difference between the proportions is large enough to conclude, at α = 0.05, that Non-Aboriginal Canadians are significantly more likely to have a university education than “Aboriginal Canadians”
- The difference between the two populations *is significant.*

9-41

Alpha levels affect Critical Region in Step 3:

ESTABLISHING THE CRITICAL REGION, ONE-TAILED TESTS VERSUS TWO-TAILED TESTS, WITH REJECTION REGION FOR ALPHA = 0.05 IN SHADE



FINDING CRITICAL Z SCORES FOR ONE-TAILED TESTS
(Single Sample Means)

Alpha	Two-Tailed Value	One-Tailed Value: Upper Tail	One-Tailed Value: Lower Tail
0.10	±1.65	+1.29	−1.29
0.05	±1.96	+1.65	−1.65
0.01	±2.58	+2.33	−2.33
0.001	±3.29	+3.10	−3.10

8-42

Significance vs. Importance

- The probability of rejecting the null hypothesis in comparing statistics is a function of four independent factors:

1. The size of the difference (e.g., means of 8.7 and 5.7 for the example above).
2. The value of alpha (the higher the alpha, the more likely we are to reject the H_0).
3. The use of one- vs. two-tailed tests (we are more likely to reject with a one-tailed test).
4. The size of the sample (N) (the larger the sample the more likely we are to reject the H_0).

9-43

Midterm, everything up to, and including last week's lecture

Does not include this week: 2 sample test, and/or Chi Square

2 hours and 50 minutes

9-44

General Instructions:

ALL EXAMS MUST BE RETURNED TO ME WITH YOUR ESSAY BOOKLET AND SCANTRON.

Read the questions carefully and be sure you are clear as to what's required before attempting to answer.

Show all of your work. Even if you produce the "wrong" answer, you can still earn some credit for your efforts. Refer to the formulas provided to help you answer the questions.

You may use your calculators, but no additional notes.

Identify the number of each problem you choose to answer.

Proportion: $p = \frac{f}{N}$ Percentage change $= \left(\frac{f_2 - f_1}{f_1}\right) \times 100$ $\bar{X} = \frac{\Sigma(X_i)}{N}$ $v = 1 - \left(\frac{f_m}{n}\right)$ $Z = \frac{X_i - \bar{X}}{s}$

Percentage: $\% = \left(\frac{f}{N}\right) \times 100$ Ratio $= f_1 / f_2$ $s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{N}}$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$

$\text{c.i.} = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{n}}\right)$ $\text{c.i.} = \bar{X} \pm Z\left(\frac{s}{\sqrt{n-1}}\right)$ $\text{c.i.} = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{n}}$

$Z(\text{obtained}) = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ $Z(\text{obtained}) = \frac{\bar{X} - \mu}{s/\sqrt{n-1}}$ $t(\text{obtained}) = \frac{\bar{X} - \mu}{s/\sqrt{n-1}}$ $Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1-P_u)/n}}$

Confidence Level	Alpha (α)	α/2	Z score
90%	0.10	0.05	± 1.65
95%	0.05	0.025	± 1.96
99%	0.01	0.005	± 2.58

Part I: Multiple Choice/True-False Questions. Please answer the following 20 questions by filling in the blank of the one best answer on the answer sheet. These will be worth 1.5 points each (30 pts total).

Part II: Problem-Solving Sets. Answer 7 out of the next 8 problems and solve them. Clearly number the problems you've chosen and show your work! Worth **10 points** per problem (**70 pts total**).